

A Study on Performance Evaluation of Some Routing Algorithms Modeled by Game Theory Approach

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Abstract

In this paper, we generally study performance of some routing algorithms. These are Dijkstra's, Bellman-Ford and Floyd Warshall algorithms that are used in shortest path problems. Further, we propose new algorithms that are modeled by game theory approach. Mathematical models have been used to solve complex problems such as those in social sciences, economics, psychology, politics and telecommunication. In this context, game theory can be defined as a mathematical framework consisting of models and techniques analyzing the behavior of individuals concerned about their own benefits. Game theory deals with multiperson decision making, in which each decision maker tries to maximize own utility or minimize own cost and is applied to networking, in most cases to solve routing and resource allocation problems in a competitive environment. Modeling the network scenarios with the game theory approach is one of the pioneering aims of the study. The algorithms for performance examination are carried out OMNeT++, which is a network simulation program. Finally, the results are compared with each other.

Keywords

Algorithm;
Performance;
Cooperative Game
Theory; Simulation;
OMNeT++.

Oyun Teorisi Yaklaşımı ile Modellenmiş Bazı Yönlendirme Algoritmalarının Performans Değerlendirmesi Üzerine Bir Çalışma

Abstract

Bu çalışmada, en kısa yol problemlerinde kullanılan Dijkstra's, Bellman-Ford ve Floyd Warshall yönlendirme algoritmalarının performansları üzerine çalışılmıştır. Ayrıca oyun teorisi yaklaşımıyla modellenerek yeni algoritmalar sunulmuştur. Matematiksel modellemeler; sosyal bilimler, ekonomi, psikoloji, siyaset ve telekomünikasyon gibi karmaşık sorunları çözmek için kullanılmaktadır. Çalışma kapsamında; oyun teorisi, kendi yararları hakkında ilgili bireylerin davranışlarını analiz eden model ve teknikleri içeren bir matematiksel çerçeve olarak tanımlanabilir. Oyun teorisi, her oyuncunun kendi faydalarını maksimize etmeye çalıştığı ya da kendi maliyetini en aza indirmeye çalıştığı çok oyunculu sistemlerde karar verme durumları için kullanılır. Özellikle yönlendirme sorunlarının ve kaynak paylaşım problemlerinin çözüldüğü rekabetçi ortamlarda ağa uygulanır. Ağ senaryolarını oyun teorisi yaklaşımıyla modellemek çalışmanın öncü amaçlarından biridir. Algoritmaların performans değerlendirmeleri OMNeT++ ağ simülatöründe yürütülmüş ve son olarak sonuçlar birbirleriyle karşılaştırılmıştır.

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Anahtar Kelimeler

Algoritma; Performans;
İşbirlikçi Oyun Teorisi;
Simülasyon; OMNeT++.

1. Introduction

A communication network is made up of nodes and links. Depending on the type of the network, nodes have different names. For example, in an IP (Internet Protocol) network, a node is called a router while in the telephone network a node is an optical or electro-optical switch. A link connects two nodes and can be called an IP link; a link in a telephone network is called a trunkgroup (Medhi and Ramasamy, 2007).

An important requirement of a communication network is to flow or route traffic from a source node to a destination node. A route is a path from the source node to the destination node. To determine a route, a routing algorithm is used. The goal of a routing algorithm is in general dictated by the requirement of the communication network and the service it provides as well as any additional or specific goals a service provider wants to impose

on itself. Thus, a network's goal generally is to address how to provide an efficient and fair routing so that most users receive good and acceptable service, instead of providing the best service to a specific user. Such a view is partly required because there are finite amount of resources in a network, e.g., network capacity (Medhi and Ramasamy, 2007).

In this paper, we consider three main algorithms that have profound impact on data networks, in particular on Internet routing. These three algorithms, known as Dijkstra's algorithm, Bellman-Ford algorithm and Floyd Warshall algorithm, are both called shortest path routing algorithms, i.e., an algorithm where the goal is to find the shortest path from a source node to a destination node. A simply way to understand a shortest path is from road networks where shortest can be defined in terms of distance, for example, as in what is the shortest distance between two cities, which consists of the link distance between appropriate intermediate places between the end cities (Medhi and Ramasamy, 2007; Reijnierse, 1996). However, it is possible that notions other than the usual distance-based measure may be applicable as well, for instance, time taken to travel between two cities. In other words, the notion of distance need not always be in terms of physical distance; it can be in other measures such as time.

We modeled these three algorithms with cooperative game theory approach and investigate the performance analysis on OMNeT++ network simulator.

2. Shortest Path Routing Algorithms

Shortest path problems lie at the heart of network flows. They are alluring to both researchers and to practitioners for several reasons. They arise frequently in practice since in a wide variety of application settings we wish to send some material between two specified points in a network as quickly, as cheaply, or as reliably as possible. They are easy to solve efficiently as the simplest network models, they capture many of the most salient core

ingredients of network flows and so they provide both a benchmark and a point of departure for studying more complex network models (Fraggelli et al., 2000). They arise frequently as sub-problems when solving many combinatorial and network optimization problems. Consequently, the study of shortest path problems is a natural starting point for introducing many key ideas from network flows, including the use of clever data structures and ideas such as data scaling to improve the worst-case algorithmic performance. Therefore, in this study, as the first step, we gather shortest path algorithms with cooperative game theory approach.

Consider a graph with nodes V and a set of edges E . Think of each node as a router and each edge as a link that can be used to send data from one router to another. Suppose we are given the transfer time c_{ij} associated with each link, and we want to get the data from node o (the origin) to node d (the destination) in minimum time. Suppose that for each $(i, j) \in E$, we take f_{ij} to be a binary variable to which we would assign a value of 1 if edge (i, j) is to be part of the route and 0 otherwise. Then, the route with shortest transfer time can be found by solving a minimum cost flow problem with $b_o = 1$, $b_d = -1$, $b_i = 0$ for $i \notin \{o, d\}$, $u_{ij} = 1$ for each $(i, j) \in E$, and an additional constraint that $f_{ij} \in \{0, 1\}$ for each $(i, j) \in E$.

Bellman-Ford algorithm uses a simple idea to compute the shortest path between two nodes in a centralized fashion. In a distributed environment, a distance vector approach is taken to compute shortest paths. The basic idea behind Dijkstra's algorithm is quite different from Bellman-Ford algorithm or the distance vector approach. It works on the notion of a candidate neighboring node set as well as the source's own computation to identify the shortest path to a destination. Another interesting property about Dijkstra's algorithm is that it computes shortest paths to all destinations from a source, instead of just for a specific pair of source and destination nodes at a time, which is very useful, especially in a communication

network, since a node wants to compute the shortest path to all destinations (Medhi and Ramasamy, 2007). The complexity of the Bellman-Ford algorithm is $O(n^3)$ while for Dijkstra's algorithm; it is $O(n^2)$.

The Floyd-Warshall algorithm, also variously known as Floyd's algorithm, the Roy-Floyd algorithm, the Roy-Warshall algorithm, or the WFI algorithm, is another algorithm for finding shortest paths in a weighted graph with positive or negative edge weights. It can also be used to detect the presence of negative cycles. A single execution of the algorithm will find the lengths of the shortest paths between all pairs of vertices, though it does not return details of the paths themselves. The Floyd-Warshall algorithm obtains a matrix of shortest path distances within $O(n^3)$ computations. The algorithm achieves this bound by applying the triple operations cleverly. The algorithm based on inductive arguments developed by an application of a dynamic programming technique (Ahuja et al., 1988). For more information we refer to (Ahuja et al., 1988; Chan, 2010; Hougardy, 2010; Kim et al., 2003).

3. Cooperative Game Theory

Game theory can be defined as a mathematical framework consisting of models and techniques analyzing the behavior of individuals concerned about their own benefits. Cooperative game theory provides analytical tools to study the behavior of rational players when they cooperate and consider the utility of all the players. An important application of cooperative games (also called coalitional games) is that they provide a mathematical formulation for collective decision-making and optimization problems. Under such circumstances, very often, the characteristic function value of a coalition can be represented succinctly as the optimum value of a combinatorial optimization problem. Such cooperative games are called combinatorial optimization games (Elliot, 2004; Reijnierse, 1996). Cooperative game theory to be associated with a variety of network types

and scenarios is seen in the literature (Ergün et al., 2014; Fragnelli et al., 200; Weber et al., 2011; Weber et al., 2008).

A cooperative n -person game in coalitional form is an ordered pair $\langle N, c \rangle$, where $N = \{1, 2, \dots, n\}$ (the set of players) and $c: 2^N \rightarrow \mathbb{R}$ is a map, assigning to each coalition $S \in 2^N$ a real number, such that $c(\emptyset) = 0$. This function c is called the characteristic function of the game, $c(S)$ is called the worth (or value) of coalition S . Often, we identify a game $\langle N, c \rangle$ with its characteristic function.

Let us clarify with simple definitions with shortest path games.

In the shortest path problems considered in this study, there is a finite set of players. Each player owns arcs, links or connections in a finite network. There are costs associated to the use of each arc. Each player receives a nonnegative reward if he/she manages to transport a good from the source of the network to its sink. In defining the class of shortest path games we rely on (Fragnelli et al., 2008).

A shortest path problem Σ is a tuple (X, A, L, S, T) , where

- (X, A) is a directed graph without loops, that is, X is a finite set, A is a subset of $X \times X$, the elements of X and A are called nodes and arcs, respectively.

- L is a map assigning to each arc is a non-negative real number $L(a)$, which can be interpreted as the length of arc a .

- S and T are non-empty and disjoint subsets of X . The elements of S and T are called sources and sinks, respectively.

Now let us introduce the relating cooperative games. There is given a shortest path problem Σ whose nodes are owned by a finite set of players N according to a map $o: X \rightarrow N$, that is $o(x) = i$ means that player i is the owner of node x . For each path P , $o(P)$ denotes the set of players who own the

nodes of P . We assume that the transfer from a source to a sink produces income g , and the cost of the transfer is given by the length of used path. A path P is owned by a coalition $S \subseteq N$, if $o(P) \subseteq S$, and we assume that a coalition S can only transfer through own paths.

A shortest path cooperative situation σ is a tuple (Σ, N, o, g) . We can associate with σ the cooperative game u_σ given as follows:

for each $S \subseteq N$:

$$c_\sigma(S) = \begin{cases} g - L_S, & \text{if } S \text{ owns a path in } \Sigma \text{ and } L_S < g \\ 0, & \text{otherwise} \end{cases}$$

where L_S is the length of the shortest path owned by S .

The shortest path game c_σ is the game associated with the shortest path cooperative situation σ (Int Ref 1).

Let us clarify the situation with an example.

3.1. An Example

Let $N = \{1, 2, 3\}$ be the set of players, the graph in Figure 1 represents the shortest path cooperative situation, $S1, S2$ are the sources, $T1, T2$ are the sink nodes.

The numbers on the arcs identify the costs (or the lengths), and $g = 7$. Player 1 owns the nodes $S1, S2, R2$, and $T2$, player 2 owns the nodes $R1, S2, R3$, and $T1$, player 3 owns the nodes $S1, S2, R2$, and $T1$; then Table 1 gives the induced shortest path game.

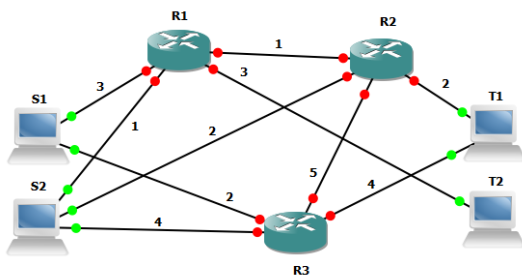


Fig. 1. The network graph of the shortest path game of an example

Table 1. The induced shortest path game of an example

S	Shortest path owned by S	$L(S)$	$u(S)$
{1}	{S2, R1, R2, T1}	4	3
{2}	{S2, R2, T1} or {S2, R1, R2, T1}	4	3
{3}	{S1, R3, T1}	6	1
{1, 2}	{S2, R1, R2, T1}	4	3
{1, 3}	{S1, R1, R3, T2}	8	0
{2, 3}	{S1, R2, R3, T1}	8	0
{1, 2, 3}	{S2, R1, R2, R3, T1}	8	0

4. The Model and Simulation Results

In order to compare the performances based on algorithm running times (which means that finding the shortest-path) of three routing algorithms and cooperative shortest game, OMNeT++ network simulator is used in this paper. OMNeT++ represents a framework approach. Instead of directly providing simulation components for computer networks, queuing networks or other domains, it provides the basic machinery and tools to write such simulations. This network simulator is a C++ based discrete event simulator for modeling communication networks, multiprocessors and other distributed or parallel systems (Int Ref 1; Varga, 2001). Since its first release, simulation

models have been developed by various individuals and research groups for several areas including: wireless and ad-hoc networks, sensor networks, IP and IPv6 networks, MPLS, wireless channels, peer-to-peer networks, storage area networks (SANs), optical networks, queuing networks, file systems, high-speed interconnections (InfiniBand), and others (Varga and Hornig, 2008). Network emulation, together with real-time simulation and hardware-in-the-loop like functionality, is available because the event scheduler in the simulation kernel is pluggable.

An OMNeT++ model consists of modules that communicate with message passing. The active modules are termed simple modules; they are written in C++, using the simulation class library. Simple modules can be grouped into compound modules and so forth; the number of hierarchy levels is not limited. Messages can be sent either via connections that span between modules or directly to their destination modules (Int Ref 1; Pintér and Radványi, 2013; Reijnierse et al., 1996; Varga, 2001). OMNeT++ is capable of animating the flow of messages on network charts and reflecting state changes of the nodes in the display. In Figure 2, some important scenes of OMNeT++ can be seen.

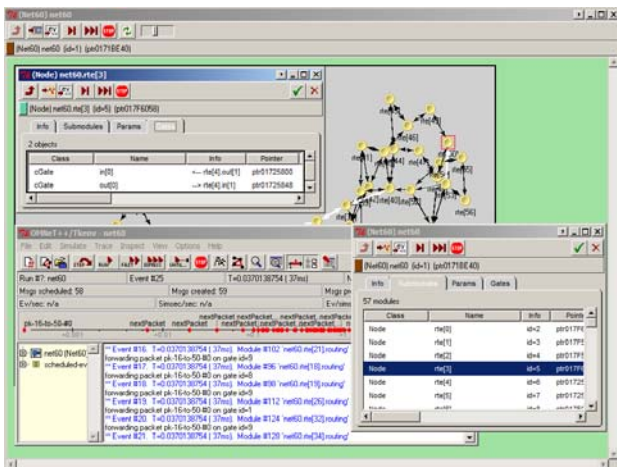


Fig. 2. Some important screenshots of the OMNeT++

Our network model demonstrates static shortest-path routing. Therefore, routing tables are set up at the beginning of the simulation using the cTopology class. Every node queries the topology

of the network independently, using a cTopology object. Then it computes shortest paths to every other node, and stores the routing parameters in a routing table. The topology is static during the simulation, and so there's no need for the nodes to do anything to keep the tables up-to-date. There's no routing protocol in the model. Once the routing tables are set up, nodes start sending packets at random intervals. Every node gets a list of destination addresses in a parameter, and for every packet it randomly chooses a destination from the list.

Our model creates a network with six nodes, generates the costs between links randomly. Herewith, we have four nodes as players, one source node and one target node.

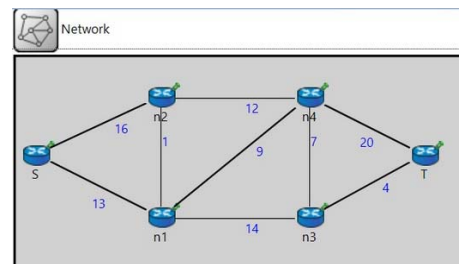


Fig. 3. Our simple network

All the algorithms prepared in OMNeT++ simulation environment. To achieve reliable results, each algorithm is performed 100 times for the network graph. To eliminate the influence of the simulation environment, extreme results are rejected and then the averages of the remaining results are calculated. The comparison of running times for the algorithms solving the shortest path problem in micro seconds is given in Figure 4.

To solve the problem of finding the shortest path, Dijkstra's algorithm gives better results than Bellman-Ford algorithm. Besides these algorithms, Floyd-Warshall algorithm remains slow. As shown, the time performances of the algorithms which are modeled by cooperative game theory approach are better than the others.

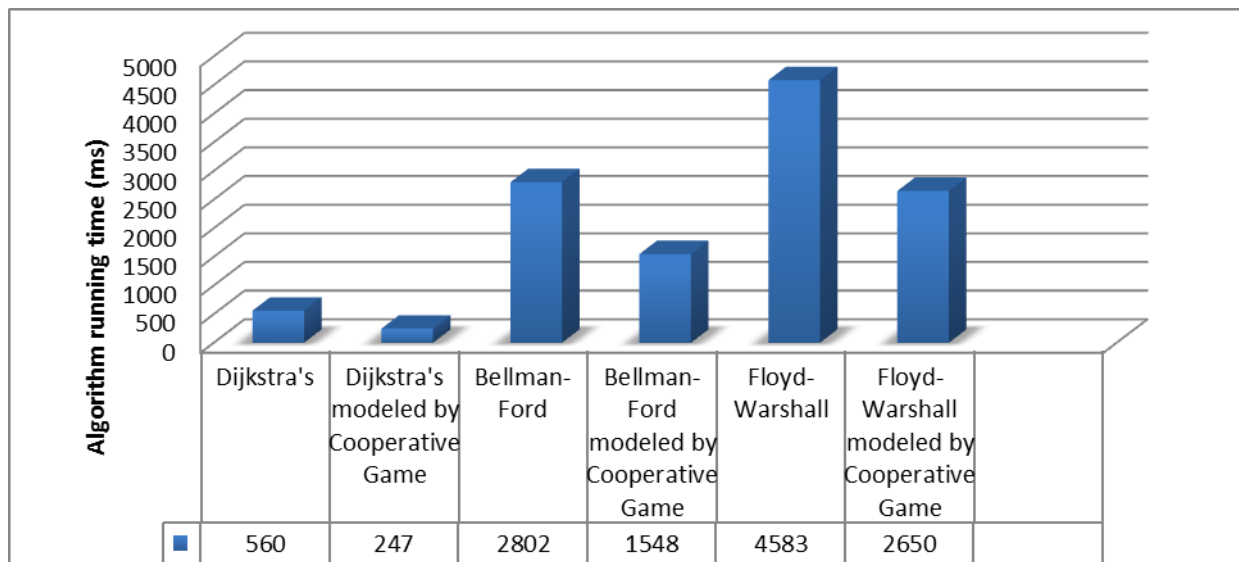


Fig. 4. The Comparisons of the Algorithms

5. The Conclusion and Outlook

This paper presented three routing algorithms solving the shortest path problem and give basic information about cooperative game theory. Performance evaluation of these routing algorithms modeled by game theory approach.

As a future work, this study can be applied to the other optimization situations such as minimum-cost spanning trees, transportation problems, maximum-flow augmenting paths, postman and related arc routing problems, the traveling salesman and related vertex routing problems, location problems, etc. in the framework of cooperative game theory. Furthermore, the simulation results this study shows that the routing algorithms can be compared with each other and the other cooperative games. A subsequent study may be the examination of these algorithms in networks of varying size.

Also, fuzzy routing, hierarchical routing, heuristic routing and geographic routing situations can be modeled with cooperative game theory, too. It is envisaged that using cooperative game theory will be bring a new approach to network performance engineering.

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